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Value of Feedback in Agricultural Decisions

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ABSTRACT

The value of feedback in the agricultural decision-making process was determined by comparison between open and closed loop optimization models under different conditions of risk aversion. An optimization via simulation of expected utility with dynamic stochastic constraints is determined using a mathematical programming routine. As a test case, shrimp aquaculture in South Texas is described by a system of stochastic non-linear differential equations. Results show that value of feedback depends on the level of risk aversion of the decision maker. Copyright © 1996 Published by Elsevier Science Ltd

INTRODUCTION

Agricultural production processes involve considerable uncertainty on the part of decision makers (DM). Output from typical agricultural processes may depend on randomness in weather fluctuations, growth and mortality rates, pests, and viruses. A DM's economic environment raises the uncertainty of input costs and output prices as well as questions regarding the supply of resources and inputs. Uncertainty associated with production processes is revealed in decision time points and consequently affects decisions by DMs (Gould, 1974) in an effort to reduce or even avoid uncertainty. Many mechanisms are available for agricultural DMs to reduce or avoid uncertainty, such as greenhouses to avoid weather fluctuations, contracting of commodities to buyers to avoid price uncertainty, extensive research and development processes to reduce uncertainty about production technologies and gathering processing and evaluating information as close as possible to real time.

Because agricultural production processes are naturally continuous, there are two types of decisions to be considered. Operational type decisions to be made and applied continuously, and scheduling type decisions in a discrete fashion (Jaquette, 1974). The interaction between operational and scheduling decisions requires special treatment when evaluating available and gathered information.

As the production process proceeds, feedback may be available and can be incorporated into the decision-making process. Then, a mechanism determining when, where and how much feedback information should be gathered by the agricultural DM is found to be attractive. Feedback is available as part of advanced and improved technologies, which are developed by agencies and governments via supported research and development (Watkins, 1991). Consequently, they are interested in evaluating feedback at a macro level.

Decision makers differ in their attitude towards risk. Higher risk aversion on the part of a DM may lead to different decisions under the same level of uncertainty (Hess, 1982). Therefore, the economic value of information may vary between DMs (Chavas & Pope, 1984; Mazzcco, *et al.*, 1992). This study focuses on economic evaluation of feedback information by comparing performance of an agricultural economic unit under biological uncertainty and dynamic consequences. The operational and scheduling questions are addressed both with and without feedback information. Both types of decisions are combined in an optimization framework and the economic interpretation of the optimization's results are discussed and analysed.

The natural variability of growth and mortality of the shrimp population in a pond provides an opportune setting for studying the value of gathering feedback information and its impact on production (Karp *et al.*, 1986; Leung, *et al.*, 1990). Decisions, such as feeding rate based on this variability are of the operational type (Clark, 1976) and decisions about when, for how long and the amount of shrimp to stock in a pond are scheduling type decisions (Talpaz & Tsur, 1982). Within the shrimp industry it is not yet clear how continuous monitoring of the production process will affect the DM's utility.

In the first section of this study a system of stochastic non-linear differential equations describes the production process. Stochastic production dynamics with respect to both operational (continuous) and scheduling (discrete) questions are also formulated. Solutions for DMs with different attitudes towards risk are examined and analysed. Solutions use mathematical and dynamic programming concepts for continuous and discrete decisions (Yaron & Dinar, 1982). This formulation allows for capturing the contribution of feedback information during the decision-making process.

The second section contains an empirical application to shrimp production in South Texas. The above problem is applied to two models: with feedback referred to as closed loop (CL) and without feedback referred to as open loop (OL).

In the third section of this investigation the value of the feedback information is examined. A run of the model with all stochastic variables assigned to their expected values is performed. Optimal solutions of this run are incorporated in a simulation of the two stochastic models allowing for different levels of risk aversions. It is found that feedback information attains higher values in the case of more risk-averse DMs. The results are provided in explicit utility units.

THE MODEL

Consider a production process that spans the period t_o to t_f. The state of the process at time t is given by the vector $\mathbf{x}(t)$ and control variables of the process are given by the vector $\mathbf{u}(t)$. There are n scheduling points (t_j, $j=1,...,n$; $t_o \leq t_j \leq t_f$; $t_j \leq t_{j+1}$) at which discontinuity in the process may occur. There is a batch of production between two consecutive scheduling points, t_{j-1} and t_j, and the process $\mathbf{x}(t)$ is assumed to follow stochastic differential equations:

$$\dot{\mathbf{x}} = f^j(\mathbf{x}, \mathbf{u}, t) + G(t)\omega(t) \tag{1}$$

where f and G are known matrices of functions and $\omega(t)$ is a white noise Gaussian variable.Other statistical properties are:

$$E[\omega(t)] = \mu_\omega$$

$$\mathrm{COV}[\omega(t), \omega(s)] = Q(t)\delta(t-s)$$

$$E[\mathbf{x}(t_o), \mathbf{x}(t_o)^T] = V_o$$

$$\mathrm{COV}[\mathbf{x}(t_o), \omega(t)] = 0$$

where Q and V are the variance–covariance of ω and $\mathbf{x}$, respectively, and δ is the Kronecker delta function (i.e. $\delta = 1$ when $t = s$, and zero otherwise).The mean pattern of $x(t)$ is

$$\dot{\bar{\mathbf{x}}} = f^j(\bar{\mathbf{x}}(t), \mathbf{u}(t), t); j = 1, ...n. \tag{2}$$

where $\bar{\mathbf{x}}(t)$ is the mean of $\mathbf{x}(t)$.Using a first or second order Taylor's series expansion around $\bar{x}(t)$, an approximation to the dynamics of the covariance pattern is:

$$\dot{V} = f_x(\bar{\mathbf{x}}, t)V + Vf_x(\bar{\mathbf{x}}, t)^T + \mathrm{GQG}^T \tag{3}$$

where $f_x(\bar{\mathbf{x}},t)$ is the Jacobean of f evaluated at $\mathbf{x}$ and superscript T stands for a transpose sign. For a single variable, Mangel (1985) referred to equations (2) and (3) as mean variance algorithms. There is a maintenance costs function $L^j\,\{\mathbf{x}(t),\mathbf{u}(t),t\}$, associated with the corresponding $f^j(\mathbf{x},t)$. At scheduling time points $(t_j, j=1,2,...,n)$, the state variables obtain different values just before and just after decisions are applied, $\mathbf{x}(t_j^-)$ and $\mathbf{x}(t_j^+)$, respectively. The actions may incur costs and create revenues as functions of the state variables and time, i.e.

$$\phi = \phi[\mathbf{x}(t_o^-), \mathbf{x}(t_o^+), , , \mathbf{x}(t_n^-), \mathbf{x}(t_n^+), t_o, , , t_n]. \tag{4}$$

Equation (4) gives strategical type costs and revenues, whereas $L^j\{\mathbf{x}(t),\mathbf{u}(t),t\}$ gives operational type costs. It is assumed that under the presence of uncertainty, a DM is interested in maximizing expected von Neumann Morgenstern utility from profits over the planning horizon.

The control optimization framework yields the objective function:

$$J(\mathbf{x}, t_o) = \max_u E\left[U\left\{\phi[\mathbf{x}(t_o^-), \mathbf{x}(t_o^+), , , \mathbf{x}(t_n^-), \mathbf{x}(t_n^+), t_o, , , t_n] - \sum_{i=1}^{n}\left(\int_{t_{i-1}^+}^{t_i^-} L^i(\mathbf{x}(t), \mathbf{u}(t), t)\mathrm{d}t\right)\right\}|\mathbf{x}(t_o) = \mathbf{x}_o\right] \tag{5}$$

subject to:

$$\begin{gathered}\psi[\mathbf{x}(t_o^-), \mathbf{x}(t_o^+), , , \mathbf{x}(t_n^-), \mathbf{x}(t_n^+), t_o, , , t_n] = 0,\\ (t_j, j = 1, , , n;\ t_o \le t_j \le t_f;\ t_j \le t_{j+1})\end{gathered} \tag{6}$$

and equations (2), (3) and (4). The set of constraints $\psi(\cdot)$ is a set of boundary conditions on the state variables and on the scheduling.When no additional information is considered to be gained as the process proceeds, an OL framework can be used with $\bar{\mathbf{x}}$ as an approximation to $\mathbf{x}$ in the function $L^i\{\mathbf{x}(t),\mathbf{u}(t),t\}\mathrm{d}t$ (Bryson & Ho, 1975). This is a non-linear constrained optimization problem and a mathematical programming approach (Gill & Murray, 1974) can be used for solving optimal values of the control variables, and the byproducts used for economic interpretation.

The CL approach is based on a set of observations and the ability to update and make decisions as information is gained. Consider a schedule of m observation time points $(z_k, k=1,2,...,m)$ in which full information about the state variables is gained and the variance of the state variables vanishes:

$$V(z_k) = 0, (z_k, k = 1, 2, , m). \tag{7}$$

At an observation time point z_k, a DM may, for the process from that time until the end of the planning horizon, revise decisions $\mathbf{u}(t)$, and change the schedules $(t_i, i=1,2,...,n)$ and $\mathbf{x}(t_i^-),\mathbf{x}(t_i^+),...\mathbf{x}(t_n^-)\mathbf{x}(t_n^+),t_i \geq z_k$.

Equation (5) is valid for any time point t with an appropriate conditioned vector $\mathbf{x}$. By conditioning on $\mathbf{x}+\mathrm{d}\mathbf{x}$ the functional $J(\mathbf{x}+\mathrm{d}\mathbf{x},t+\mathrm{d}t)$ can thus be found. Moreover, the time path (t,t_f), on the left hand side of equation (5) can be split into time path $(t,t+\mathrm{d}t)$ and $(t+\mathrm{d}t,t_f)$. If the utility function $U[\cdot]$ is separable, so that it can be a function of $J(\mathbf{x}+\mathrm{d}\mathbf{x},t+\mathrm{d}t)$, then by extrapolating from equation (5):

$$J(\mathbf{x}, t) = \max_u E\left\{ H[\phi(\cdot) - \int_t^{t+\mathrm{d}t} L(\cdot)\mathrm{d}s, J(\mathbf{x} + \mathrm{d}\mathbf{x}, t + \mathrm{d}t)] | \mathbf{x}(t) = \mathbf{x}_t \right\} \tag{8}$$

where $\tilde{\mathbf{x}}$ is an approximation to $\mathbf{x}$ in the functional $L^i\{\mathbf{x}(t),\mathbf{u}(t),\mathrm{t}\}\mathrm{d}t\}$, and $H[\cdot]$ is a transformation of $U[\cdot]$. At time point z_k, the value of $\mathbf{x}$ is known, yet $\mathrm{d}\mathbf{x}$ is a random vector.Equation (8) serves as the recursive functional in dynamic programming terminology. Boundary conditions are $\phi[\mathbf{x}(t_f)]$; the revenues from a state level $\mathbf{x}$ on the last day of the planning horizon.

Finding an analytical solution to equation (8) is a difficult task and is possible only in the case of specific functional forms and particular probability distributions. The recursive property of equation (8) and Bellman's Optimality Criteria can be used in order to solve backward from time point z_m until z_1. For any observation time point z_k, various $J(\mathbf{x},z_k)$ are computed for different $\mathbf{x}$, and an interpolation is made in order to get a functional form of $J(\mathbf{x},z_k)$ with respect to the vector $\mathbf{x}$. The functional $J(\mathbf{x},z_k)$ are introduced into equation (8) for solving for $J(\mathbf{x},z_{k-1})$. This procedure is repeated until z_1.

Two objective functions have been determined. Equation (5) acts as the objective function in the case of an OL framework with no feedback. Equation (8) is extrapolated from equation (5) to act as the objective function in the CL framework with available feedback.

APPLICATIONS

The model can be applied to many agricultural branches in which a production over time is associated with uncertainty. Operational type decisions can be updated as a result of gaining information about the production process. For example, irrigation of field crops with low quality water, feeding milk cows, preparing silage for feeding livestock, etc. In these cases, research and development processes may result in production technologies with less uncertainty. The economic motivation to conduct such effort can be quantified by this model. The case study of this work is based on a branch of aquaculture due to the ability to transfer production technologies and know-how from one place to another, by modifying the relevant, functional forms and parameters.

OL and CL decision models are applied to the management of a shrimp farm in Texas. The growing season of shrimp in ponds is limited to 33 weeks a year from early April to late November due to weather conditions. Management problems faced by DMs include: how many crops to produce per growing season (scheduling), size and amount of shrimp to stock ($\mathbf{x}(t_i^+)$), and the optimal rate of feeding ($\mathbf{u}$) (operating). Scheduling point decisions ($t_i, i=1,2,...,n$) are made at the beginning of a growing season and include number of crops, amounts to stock and harvest times. Operational decisions include the feeding of the shrimp in ponds.

Feedback for the decisions may be obtained by observing the ponds on decision days. It is assumed that experienced farmers use the information from the observations to estimate precisely the biomass in the pond $\mathbf{x}(t_i^-)$. In this study, observation days are every seven weeks; that is, $z_1=7$, $z_2=14$, $z_3=21$, and $z_4=28$. On every observation day (the last day of a week), scheduling of up to five additional crops is tested from that day till the end of the growing season ($j=1,2,,,5$). These parameters and options are chosen because they are relevant for this case study. The number of observation points of time and the corresponding number of decisions can easily be increased.

Functional forms

The vector $\mathbf{x}$ has two components: w_t, the size in grams of an individual shrimp and q_t, the quantity of shrimp in a 1 acre pond. There is randomness around shrimp growth rate and mortality rate over time; consequently, biomass level on a given day is not known with certainty.

The function $f(\cdot)$ from equation (1) is built from a two-component vector:

$$\dot{\mathbf{x}} = [\dot{w}, \dot{q}]$$

$$\dot{w}_p = w_p cbe^{(-ct)}e^{(dwq)}$$

$$\dot{q} = -he^{(-fw)} + kq$$

$$\dot{w} = g\dot{w}_p \tag{9}$$

where $\dot{w}_p$ is the potential rate of growth, b, c, d, f, h, and k are coefficients and g is a number between 0 and 1 describing the desired growth rate.-

Variable costs associated with maintenance and growth of shrimp mostly consist of the costs of feeding:

$$L[\mathbf{x}, \mathbf{u}, t] = q(t)\mathrm{HT}(w^{0.75} + l\dot{w})(Pf/\mathrm{CI}) \tag{10}$$

where Pf is unit price of feed and CI is a conversion coefficient index from kilo-calories to kilograms of feed. The parameter l is the impact of shrimp weight gain on energy intake; HT is the impact of temperature on energy intake and $q(t)$ is the number of shrimp. The costs of juvenile shrimp for stocking are fixed for the farmer. The market price of shrimp is related to its size and season of the year. The impact of time on the unit price is crucial because shrimp size is proportional to its time in the pond and the unit price goes up accordingly. The price of an individual shrimp is a function of season and shrimp size:

$$h(w) = v\left[1 + \frac{w_o^p - w^p}{e^{-stw^p}}\right]^{-1} \tag{11}$$

where time zero is the beginning of April. The parameters are v, p, s and w_o. More information on the functions of the applied case can be found in Sadeh (1986). Dynamics of the variance–covariance of $\mathbf{x}$ are captured by

$$G(t) = Gt^{0.5} \tag{12}$$

where G is a matrix of coefficients. The impact of time on the variance is on the increase but at a decreasing rate.The parameters specified by equations (3) and (9) were estimated using experimental data from six ponds in South Texas[1]. The implied growth path was subjectively judged to be representative by industry experts.

[1]The estimated coefficients are: $a = 43.3$, $c = -0.094$, $h = -253.0$, $f = -0.014$, $k = -0.0063$, $h_{11} = 0.102$, $h_{12} = 0.236$ and $h_{22} = 0.586$. Where a is an asymptotic weight of shrimp and it is given in a solution of the differential equations. The coefficient b is calculated based on the other coefficients. More about the data and the estimation procedure can be found in Sadeh (1986).

For operating type decisions a farmer assumes that biomass is at its expected value and feeds shrimp accordingly. The impact of the random variable, biomass to be harvested from ponds ($\mathbf{x}(t_i^-)i = 1,...,n$) is considered on scheduling days. Consequently, although the state variables $\mathbf{x}(t_i)$ are subject to uncertainty, the $\mathbf{x}(t)$ in the loss (cost) function are considered to be at their expected values.

A risk-neutral DM maximizes the expected profits from a pond. Yet, a von Neumann Morgenstern utility function with an exponential functional form is chosen to describe the farmer's attitude towards increasing uncertainty as

$$U(\pi) = b(1 - e^{-a\pi}). \tag{13}$$

For the OL model, equation (5) gives the optimization rule with respect to control variables **u**, where **x** is the random variable for which the expectation operation is referred. The value of **a** in the exponential utility function was set by inspecting many values of **a** in the interval [0·000009, 0·01], and the parameter **b** = 1000 was chosen arbitrarily. For relatively large values of **a**, the exponential utility function is concave; it reaches the asymptotic line **b** quickly. The optimal objective value for an OL model, with respect to increasing **a**, is a unimodal function getting its maximum value (79·5) at **a** = 0·000045 and minimum value zero ("do nothing"). For small values of **a**, one crop is found to be the optimal policy. At **a** = 0·000098, a two crop policy is optimal, whereas for **a** > 0·00014 "do nothing" is the optimal policy. Along with a risk-neutral DM situation, two different values of **a** were chosen to represent the effect of risk aversion of the DM: **a** = 0·00005, and **a** = 0·0001.The recursive equation for this CL model is given by substitution into equation (8):

$$J(\mathbf{x}, t) = \max_{u} E[b(1 - e^{-a\pi_d} + J(\mathbf{x} + \mathrm{d}\mathbf{x}, t + \mathrm{d}t)e^{-a\pi_d}] \tag{14}$$

where π_d is the profit gained from time t up to $t + \mathrm{d}t$, and $J(\mathbf{x} + \mathrm{d}_x, t + \mathrm{d}t)$ is the optimal value for $\mathbf{x} + \mathrm{d}\mathbf{x}$ from $t + \mathrm{d}t$ until the end of the planning horizon. At time t, the variables π_d and $(\mathbf{x} + \mathrm{d}\mathbf{x})$ are random and the operator $E[\cdot]$ is related to them.Bellman's optimality principle is applicable here. Consequently, the dynamic programming approach and the convergence of the optimum algorithm for separate decisions are assured. At a given point of time t, the state variable $\mathbf{x}(t)$ is known with certainty, and a DM takes the best action regardless of what occurred prior to time t. If the function $J(\mathbf{x},t)$ is concave with respect to the decision variable (**u**), then the optimal solution can be found regardless of previous decisions.

At time $t + \mathrm{d}t$, $J(\mathbf{x} + \mathrm{d}\mathbf{x}, t + \mathrm{d}t)$ was found for many possible $\mathbf{x} + \mathrm{d}\mathbf{x}$ values, and a quadratic form was used to obtain a smooth functional form for all other possibilities that have not been simulated:

$$J(\mathbf{x} + \mathrm{d}\mathbf{x}, t + \mathrm{d}t) = a_0 + a_1(\bar{w} + \mathrm{d}w) + a_2(\bar{q} + \mathrm{d}q) + a_3(\bar{w} + \mathrm{d}w)^2 + a_4(\bar{q} + \mathrm{d}q)^2 + a_5(\bar{w} + \mathrm{d}w)(\bar{q} + \mathrm{d}q). \quad (15)$$

The expectation part in equation (14) can be approximated by a Taylor's series expansion around $E[\pi_\mathrm{d}]$ and $E[\mathbf{x} + \mathrm{d}\mathbf{x}]$. It contains three parts: (i) its evaluation at the mean of π_d and $(\mathbf{x} + \mathrm{d}\mathbf{x})$; (ii) its second derivative with respect to π_d multiplied by half of the variance of π_d; and (iii) its second derivative with respect to $(\mathbf{x} + \mathrm{d}\mathbf{x})$ multiplied by half of the variance of $(\mathbf{x} + \mathrm{d}\mathbf{x})$. There is no correlation between π_d and $(\mathbf{x} + \mathrm{d}\mathbf{x})$. The approximation of equation (14) is:

$$E[\cdot] = b(1 - e^{-a\bar{\pi}}) + J(\bar{\mathbf{x}} + \mathrm{d}\bar{\mathbf{x}}, t + \mathrm{d}t)e^{-a\bar{\pi}} + a^2 e^{-a\bar{\pi}}[J(\bar{\mathbf{x}} + \mathrm{d}\bar{\mathbf{x}}, t + \mathrm{d}t) - b] \cdot 0.5\mathrm{Var}(\pi_\mathrm{d}) + e^{-a\bar{\pi}}(a_3 V_{11} + a_4 V_{22} + a_5 V_{12}) \quad (16)$$

where V_{ij} are elements in the computed variance–covariance matrix of vector $(\mathbf{x} + \mathrm{d}\mathbf{x})$. The second derivative of $J(\mathbf{x} + \mathrm{d}\mathbf{x}, t + \mathrm{d}t)$ is easily interpreted from the quadratic approximation of $J(\mathbf{x} + \mathrm{d}\mathbf{x}, t + \mathrm{d}t)$ provided above.The computation of approximation to the expected value of π, as well as the variance of π, are given in Sadeh (1986). To avoid numerical difficulties, several conditions are set during simulation and optimization. It is always required that

$$\frac{\partial}{\partial \pi}\{U(\bar{\pi}) - E[U(\pi)]\} \geq 0 \quad (17)$$

which is satisfied by the assumption of concavity of $U(\pi)$. Also, the variance covariance matrix should be positive definite: $V_{11} > 0$, $V_{22} > 0$ and $V_{11}V_{22} - V_{12}{}^2 > 0$.

RESULTS

Seven runs (i–vii) with variation in levels of risk aversion and presence of feedback are presented in Table 1. Table 1 also contains the optimal number of crops, the optimal number of crops per growing season, the optimal values of decision variables, their corresponding shadow prices and objective values. Values of selected variables on harvest days are presented in Table 2.

TABLE 1

Optimal decisions and their corresponding shadow prices by feedback and risk aversion levels

Run number	Risk aversion coefficient a	OL/CL	Crop no.	Optimal decision values				Shadow prices(utility units)				Objective value(utility units)	Simulation of a stochastic run (utility units)
				TH(weeks)	w(g)	q('000s/pond)	g(ratio)	TH	w	q	g		
i	CE		1	33·0	1	60	1	177·0	499·0	83·0	9553	6259·0	6259·0
ii	Risk neutral	OL	1	33·0	1	60	1	130·2	1456·3	104·3	12438	4255·0	4255·0
iii	Risk neutral	CL	1	33·0	1	60	1	190·0	499·3	81·6	9557	6383·0	6383·0
iv	0·00005	OL	1	29·7	1	60	1	0·0	103·6	3·7	614	78·5	74·9
v	0·00005	CL	1	33·0	1	60	1	1·6	14·4	3·1	315	252·2	252·2
vi	0·0001	OL	1	16·2	1	60	1	—	125·0	0·3	630		
			2	33·0	1	60	1	—	126·0	0·2	624	32·3	−30·3
vii	0·0001	CL	1	30·6	1	60	1	0·0	30·3	4·3	519	418·0	410·6

TH is time of harvest in weeks from stocking day.
w is stocking size of an individual shrimp in grains.
q is stocking density in thousands of shrimp per pond and
g is a ratio of actual growth rate to potential growth rate.

TABLE 2
Values of selected variables on harvest days by feedback and risk aversion levels

Run number	Risk aversion coefficient **a**	OL/CL	Crop no.	Ending values on harvest day(s)						
				Biomass(kg/pond)	w(g)	q('000s/pond)	Total feed(kg/pond)	Var(w)	Cov(w,q)	Var(q)
i	CE		1	1093	28·1	38·8	3091	—	—	—
ii	Risk neutral	OL	1	1093	28·1	38·8	3091	241·8	33·3	914·0
iii	Risk neutral	CL	1	1093	28·1	38·8	3091	9·2	16·6	339·0
iv	0·00005	OL	1	1047	25·8	40·6	2620	215·0	34·8	715·0
v	0·00005	CL	1	1093	28·1	38·8	3091	9·2	16·6	339·0
vi	0·0001	OL	1	655	13·5	48·5	1010	64·6	23·4	172·5
			2	636	13·0	48·8	944	59·5	22·3	161·4
vii	0·0001	CL	1	1061	26·5	40·1	2744	4·5	9·3	168·0

Biomass is kg of shrimp per 1 acre pond.
w is stocking size of an individual shrimp in g.
q is stocking density in thousands of shrimp per pond.
Total food is the accumulated kg of food given per 1 acre pond per crop.
Var(w) is the variance of w on harvest day.
Var(q) is the variance of q on harvest day and
COV(w,q) is the covariance between w and q.

The decision variables stocking size, stocking density and feeding policy are at their upper boundary values in all runs. Time of harvest, however, generally varies between runs. Results therefore, are compared in the number of crops, the cultivation length of these crops and the corresponding shadow prices of decision variables.

The first optimization run was performed on the current problem with all random variables assigned to their expected values, with no variance considered. Under conditions of run (i), one crop was found to be the optimal solution. The objective value is 6259 utility units and the crop is cultivated for the entire growing season (33 weeks).

Risk neutral

Under risk neutral assumptions, an OL model was run (ii) and expected utility is maximized. This particular run results in 4255 utility units and with decision variables at their boundary level. A positive covariance between the size and number of shrimp has positive impact on expected profits, whereas the variance of shrimp size has the opposite impact. Because the impact of the covariance is greater than the impact of the variance of size, the optimal harvest day is at the upper bound of the growing season constraint.

Under the economic conditions stated above, the decision variables stocking size, stocking density, feeding policy and harvest time have a positive impact on the general economic performance of one shrimp pond. Yet, the first three factors contribute less uncertainty than the harvest time by increasing the cultivation period of shrimp. Therefore, the tendency to increase output under uncertainty has more pressure towards increasing stocking size and stocking density of shrimp as it is revealed by the shadow prices.

A risk-neutral DM will take the most risky prospect under the conditions of this run. That is, to have a single crop with the maximum growing period without additional information or feedback during the growing period.

When feedback is available a CL model was run (iii) with resulting small variance–covariance of state variables. Therefore optimal decisions are at their upper bounds as in run (i). The objective value is higher (6383 utility units) than in the OL model run (ii). The shadow price of stocking size is (499·3) smaller than (1456·3) of the OL model run (ii). The opposite holds for the harvest day. The tendency to produce leads DM more towards increasing initial size than increasing the cultivation period of a crop. Under the conditions of run (iii) the uncertainty is much smaller than uncertainty resulting under run (ii) conditions, and there is more pressure towards increasing the growing season. Increasing the growing season is possible with different technologies such as greenhouses, and consequently requires more investment.

Low risk aversion

For a DM with risk coefficient **a** = 0·00005, one crop is optimal in both the OL and CL model runs (iv and v). The one-crop policy resulting from the OL (iv) is sub optimal to the solution of run (i). Because the crop is scheduled for only 29·7 weeks rather than the entire growing season of 33 weeks. This change in the scheduling is due to the variance gained in the last few weeks of the crop and the level of risk aversion.

The solution of CL model run (v) is similar to the run (i) solution. The expected full information from the four observation days is very close to conditions of certainty under these run conditions.

A DM under OL model conditions (iv) can produce only one crop in a growing season, but with 29·7 weeks cultivation period. The contribution of the information to be gained under CL model conditions (v) allows DMs to expand the cultivation period from 29·7 to the maximum possible length of 33 weeks.

The objective value in run (iv) is 78·5 utility units and 252·2 utility units in run (v). As mentioned above, this is consistent with *a priori* expectations. The lower variance and thus longer cultivation period in run (v) account for the increased utility units. For the risk averse DM, feedback information enhances utility.

High risk aversion

A DM with risk coefficient **a** = 0·0001 is more risk-averse, and the role of variance is very crucial. In the OL model run (vii), the DM prefers a two-crop policy with less variance at the end of each crop than one crop with higher variance (Table 2). A high risk averse DM cannot bear the uncertainty associated with a single crop and he would rather split the growing season into two short cultivation periods of about 16 weeks each. Because the two-crop policy is inefficient under conditions of run (i), the DM is ready to offset economic efficiency by not bearing uncertainty.

Under the CL model run (vii) the DM prefers a single-crop policy to a two-crop policy. However, the cultivation period is 2·4 weeks short of the 33 weeks of run (i). The effect of increased variance shortened the length of the cultivation period.

The shadow price of stocking size is greater in the OL model run (vi) than in the CL model run (vii). The opposite is true for the shadow price of stocking density. The two-crop policy in (vi) does, however, increase the need for large initial shrimp sizes due to the shorter growth period per crop. In the case of stocking density, the opposite occurs, due to the length per crop, the shadow prices of number of shrimp are smaller in run (vi) than in run (vii).

It is interesting that, in the OL model run (vi), the biomass of shrimp in a two-crop policy totals to 1291 kg, which is more than the biomass of the CL model run (vii) (1061 kg) and more than the biomass of run (i) (1093 kg). The quality of shrimp, however, is judged by size. Shrimp size, as well as biomass, is an important factor affecting the objective function. There is a sharp decrease in actual shrimp size upon marketing when the season is split into two crops (vi). Although, in general, farmers prefer less product with higher quality, a highly risk-averse DM will give up quality for less variance.

Value of information

The models developed previously can be used to determine the value of information concerning the distribution of random variables associated with shrimp growth and mortality. The methodology of calculating such values is based on an assumption that although being in a stochastic environment and not knowing the distributions, a DM repeatedly applies optimal decisions of a run where only expected values are considered. Knowledge of the amount that a DM is willing to pay for knowing the distribution of the random variables may lead to better research and development programmes, and relevant efforts concerning learning curves of new industries. It is expected that the attitude of a DM towards risk and the nature of technology (with and without feedback) will affect the amount that the DM is willing to pay for information.

In order to determine the effect of lack of feedback information, optimal decisions from run (i) are applied in a stochastic model, and a path is simulated. This simulated path gives a scenario of having a stochastic environment without knowing the distribution behind its random variables. The difference between sub-optimal results from the simulations and those obtained by stochastic optimization can be interpreted as the value of information of knowing the distribution of random variables in a stochastic environment. This method is a special case of treating a certainty equivalence model as a case of no uncertainty (Karp *et al.*, 1986).

These differences provide grounds for comparative study. Such a system allows us to compare the relative losses due to lack of feedback information on the state of the random variables. Contrasting relative losses in both the OL and CL models gives a more accurate picture of the value of information than the use of unadjusted objective function values. Results of this method of information evaluation are found in Table 1 in the column: simulation of a stochastic run.

There is no loss of utility for a risk-neutral DM in runs (ii) and (iii), as the decisions he applies in the stochastic OL and CL models are the same as those in run (i). This is not a general conclusion, but a result for these particular runs.

For a low risk-averse DM, very small losses do occur by applying optimal decisions of run (i) in the stochastic environment. This is consistent with *a priori* expectations, as the differences between the values of the decision variables under the low-risk producers are very similar to those of run (i). The slight lengthening of the cultivation period, and thus increased ending variance under run (i) rules accounts for the utility loss.

Runs for the highly risk-averse DM yield the most interesting results. When optimal decisions of run (i) are applied in a stochastic environment without feedback, −30·3 utility units are incurred compared to +32·3 units of run (vi). This relatively large loss is clearly due to the difference between one-crop and two-crop policies as recommended by run (i) and run (vi), respectively. The difference, 62·6 utility units, is interpreted as the amount of utility that a DM is willing to give up for knowing the probability distribution of random variables in the model.

Using optimal decisions of run (i) in the CL model run with a highly risk-averse DM results in a slight loss in utility (7·4 = 418·0−410·6). The reason for the low magnitude of this loss is due to the small deviation between results of run (i) and results of run (vii) as computed for these particular runs.

Conclusions of the results

As per *a priori* expectations, the contrast between the OL and CL models is shown in the increasing objective function values (Table 1) and decreased variance levels of the state variables under CL model runs with feedback (Table 2). With respect to OL models, decreased variance in the CL models leads to a lengthening of the cultivation period for risk-averse DMs. The decisions relative to time of harvest (TH) varies among the runs (Table 1).

Risk aversion has a strong impact on shrimp aquaculture. The impact of variance is substantial. Boundary conditions on the length of the growing season yield positive but rather small shadow prices for a risk-neutral DM. For a risk-averse DM, the suggested cultivation period is less than the maximum possible. In preliminary runs of the models, it is found that under extreme levels of risk aversion (risk coefficients greater than 0·00014) a DM chooses not to produce.

Under the assumption of a highly risk-averse DM, there is a switch to a two-crop policy for the OL model run (vi). In this case, the CL model run (vii) shows a significantly higher value for the objective function. This is due to the high concavity of the utility function and the massive reduction in variance in the CL model runs with feedback.

Shrimp quality also has an impact on decision making under uncertainty. It was found that a DM agrees to bear more risk in order to have a better quality product. For example, in the one-crop scenario (Table 2; vii) individual marketable shrimp are 26·5 g and the total marketable biomass 1061 kg vs. 13·5 or 13 g marketable shrimp and 1291 kg marketable biomass in a two-crop scenario (vi).

Simulation of optimal decisions from run (i) in a stochastic environment reflects the value of knowledge of the system's probability distribution of random variables. The findings show that the DM's willingness to pay for information on the random variables corresponds to his level of risk aversion. Major differences, however, came through different policies rather than through different tactical decisions, e.g. differences in time of harvest.

SUMMARY

The model was developed as a tool to encompass operational and management decisions in agricultural systems under conditions of uncertainty. A DM facing production uncertainty over time has the alternative to update decisions as information is gathered over time. The case study is shrimp aquaculture, but with appropriate modifications it can be applied to different disciplines in agriculture.

The model is applied for five decision points of time in this case study. Increasing the number of decision points of time is easily applicable with moderate increase of computation time for relevant problems.

An economic interpretation of the results is attractive and possible due to the mathematical programming formulation of the model. The results are quantified in terms of utility units. It is found that a risk-averse DM tends to split production processes in order to avoid uncertainty. In shrimp aquaculture this means increasing the number of crops per growing season and shortening the cultivation period of each crop. A run with random variables at their expected values provides the ground for optimal riskless decisions. The difference in optimal decisions between the various risk-averse DMs with respect to that run's results means that some DMs may be inefficient and therefore may not enter the shrimp industry.

The information gathered as the production process proceeds is evaluated. The implications of the value of feedback information can be used

in evaluating possible investments in agriculture. An individual DM willing to pay money or to give up utility units for knowing the distributions of random variables associated with agricultural production processes, may lead to better research and development programmes and relevant efforts concerning learning curves of agricultural industries.

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